Algorithmic Bioinformatics I: Exercises
Assignment 5

Deadline: Wednesday, 03.06.2009, 10 ct, Amalienstr 17, Room 306

ATTENTION: As there are no lectures on Tuesday (02.06.09), the exercise sheets have to be handed in or submitted by Email until Wednesday (03.06.09) 10 ct directly to Caroline Friedel (Amalienstrasse 17, Room 306).

Exercise 1 (Recurrent Relations):
Solve the following recurrent relations for $N \geq 2$. In subitems b) - d) restrict to powers of two, i.e. $N = 2^n$. (Keywords: substitution, geometric progression, series)

(a) $C_N = C_{N-1} + N$, $C_1 = 1$
(b) $C_N = C_{N/2} + 1$, $C_1 = 0$
(c) $C_N = C_{N/2} + N$, $C_1 = 0$
(d) $C_N = 2C_{N/2} + N$, $C_1 = 0$ (typical for divide and conquer).

Exercise 2 (Master Method):
State strict asymptotic borders for the following recurrences:

(a) $T(n) = 4T(n/2) + n$
(b) $T(n) = 4T(n/2) + n^2$
(c) $T(n) = 4T(n/2) + n^3$

Use the master method and prove the choice of the right case.
Exercise 3 (GCD, Recursion):
Consider the problem of “reducing a fraction to its reduced form”. E.g. we want to reduce 4/6 or 200/300 to 2/3. The problem is equivalent to the determination of the greatest common divisor of nominator and denominator. A solution to this problem was already proposed by Euclid and is based on the fact that given two natural numbers $u$ and $v$ where $u > v$, the GCD of $(u, v)$ is equal to the GCD of $(v, u - v)$.

(a) Design a recursive algorithm for the given problem based on Euclid’s observations.

(b) Design an iterative variant of this algorithm.

Provide commented pseudocode for both cases.

Exercise 4 (Algorithm Design):
A sorted array $S$ of $n$ natural numbers is given as well as an additional natural number $x$. Design an algorithm with runtime in $O(n)$ which determines whether there are two elements in $S$ which sum up to $x$. Provide commented pseudocode and analyze its runtime.